High-Temperature Deformation of Two Phase Structures [and Discussion]

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High-temperature deformation of two phase structures

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Some of the ways in which two phase crystalline substances deform are reviewed. By the addition of a small amount of a finely dispersed second phase the resistance of a pure material to deformation can be spectacularly increased. The effect is quite disproportionate to the amount of second phase added. It stems from the fact that crystal dislocations have to increase in length to circumvent the particles and the energy needed to produce this effect must be supplied by the external machine causing deformation.

If equal amounts of equally deformable phases are present (as in certain alloys of eutectic or eutectoid composition) then the material has a low resistance to deformation but a very high ductility: the phenomenon of superplasticity. Deformation now occurs by sliding at the interphase boundaries (i.p.bs). Superdislocations whose glide causes this sliding are effectively trapped in the boundary – a condition of thermodynamic equilibrium which (it is shown) gives rise to the observed deformation properties.

At higher deformation rates dislocations appear in the grains (particles) and may form into cells. They are not, however, trapped in the cell boundaries and are observed to produce strain by traversing the cells. The aggregate now exhibits the same deformation as the cells of which it is composed. The cell size is that which minimizes the free energy for a given rate of deformation and the ductility is high, but not so high as that produced in the superplastic state.

A new mechanism of deformation appears if transformation continues during straining. Large, self-cancelling internal stresses and strains occur and the external strain rate polarizes the latter, producing external strain. The external strain rate bears the same proportionality to the external stress as does the internal strain rate to the internal stress. An analogy is provided by irradiation creep where crystallization of self-interstitials created by bombardment occurs continuously from the supersaturated solid solution. Here the rate of transformation and the internal stress have actually been measured and the predicted creep characteristics correlate well with in-pile creep data.

INTRODUCTION

Much of the existing fabric of our knowledge, both theoretical and experimental, about the deformation of crystalline materials derives from pure substances. By contrast, many commercial alloys and naturally occurring minerals comprise two or more phases. In this paper I review some of the modes of deformation for two phase materials paying special attention to areas where recent advances in understanding have occurred.

The simplest case might be thought to be that in which a very small amount of one of the phases is present and so we shall start with this. In the event, we shall see that despite its superficial simplicity this case has many interesting and unexpected features, most of them stemming from the properties of the crystal lattice.

DISPERSION HARDENING

The simplest hypothesis about the effect of a second phase upon deformability is that Einstein's (1911) equation would apply:

$$\eta_c = \eta(1+2.5\,C_p), \tag{1}$$

where C_p is the volume fraction of dispersed second-phase particles and η is the viscosity of the medium in which the particles are dispersed. This equation predicts that the viscosity, η_c, of a dilute suspension is a linear function of the amount of second phase which is dispersed. As C_p approaches zero, the effect of the dispersed phase on viscosity becomes vanishingly small.

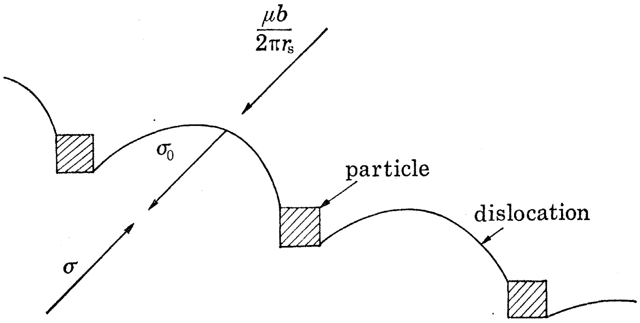


FIGURE 1. Dislocation held up by dispersed particles and subject to the external stress (σ), a friction-stress due to the particles (σ_0) and a stress due to neighbouring dislocations ($\mu b/2\pi r_s$).

Dilute suspensions of particles in liquids obey equation (1) quite well. However, dispersion-strengthened crystalline materials do not seem to obey it at all. For example, consider the case of silver containing a dispersion of fine alumina particles (Oguchi, Oikawa & Karashima 1974): here the addition of only 0.05 % of the dispersed particles (i.e. $C_p = 5\times10^{-4}$) was enough to introduce a threshold stress estimated by Evans & Knowles (1976) to be 29 MPa. Below this stress deformation occurred very slowly. By contrast equation (1) would predict a negligible effect of such a small concentration of particles upon deformability. The reason for this difference is that, in a fluid, flow will occur around any small cluster of atoms whose deformation is impeded by a particle, since the flowing atoms are not on a permanent lattice. By contrast, in a crystal, the atoms are constrained to remain on their lattice sites by large interatomic forces. A particle which hinders deformation in one part of the crystal therefore prevents movements of contiguous regions too.

From an analysis of the interaction between particles and dislocations in crystalline materials the theoretical equation for the creep of dispersion strengthened crystalline materials has been derived (see Gittus 1975*a*).

Thus a dislocation is subjected to the stress ($=\mu b/2\pi r_s$) due to its neighbours and to stress σ due to an externally applied force (figure 1). If the specimen contains a dispersion of impenetrable obstacles then the condition for the dislocation to glide is that the difference between the external stress and the internal stress due to the other dislocations must equal or exceed a friction stress σ_0:

$$\sigma - \mu b/2\pi r_s \geqslant (\mu b/xL = \sigma_0).$$

Here L is the distance between adjacent particles, μ is the shear modulus, r_s is the spacing between adjacent network dislocations (the dislocation density ρ then equals $1/r_s^2$), b is the Burgers vector and x is a factor of order unity whose exact magnitude depends on the details of the mechanism whereby the dislocation penetrates the array of dispersed particles (Brown & Ham 1971). Following the lines of the analysis detailed in previous papers (Gittus 1974*a*, *b*) then the steady-state creep rate ($\dot{\epsilon}$) should, theoretically, be

$$\dot{\epsilon} = (8\pi^3 c_j)\frac{D_v \mu b}{kT}\left(\frac{\sigma}{\mu}-\frac{b}{xL}\right)^3; \tag{2}$$

c_j is the jog concentration, D_v is the volume self-diffusion coefficient at temperature T(K) and k is Boltzmann's constant.

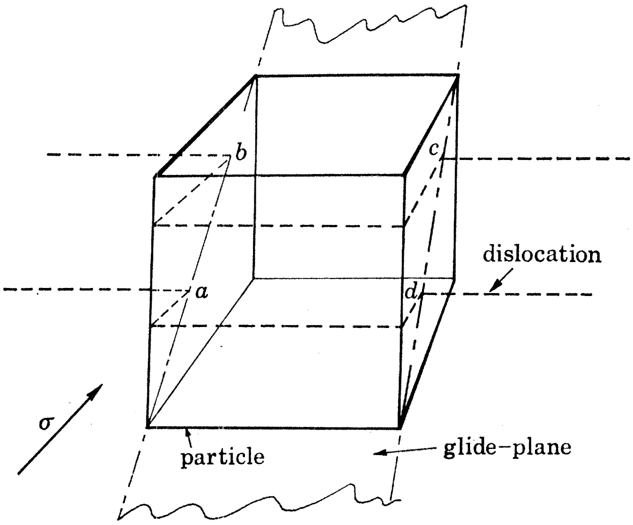


FIGURE 2. Brown & Ham's (1971) model of a dislocation surmounting a cubic particle. The glide plane is *abcd*. As the dislocation moves from *ad* to *bc*, additional line length must be created.

If the material contains no dispersed particles, then equation (2) reduces to that for a pure material developed in the earlier work (Gittus 1974*a*), as it should. The condition for creep to occur is that the external stress should exceed $\mu b/xL$. The dispersion-strengthened material creep-characteristics are in fact simply those of a pure material subjected to a smaller stress:

$$\dot{\epsilon}(\text{disp}, \sigma) = \dot{\epsilon}(\text{mat}, \sigma-\sigma_0). \tag{3}$$

In equation (3), $\dot{\epsilon}(\text{mat}, \sigma-\sigma_0)$ is the creep rate of the matrix material in the absence of a strengthening dispersion and under an external stress of magnitude $\sigma-\sigma_0$; and $\dot{\epsilon}(\text{disp}, \sigma)$ is the creep rate of the disperson-strengthened material at a stress σ. Equation (3) offers us an alternative method of estimating the creep resistance of a dispersion strengthened material if we have creep data for the pure matrix: in the absence of such data we use equation (2).

As an example of the absolute magnitude predicted by equations (2) and (3) we consider the dispersion-strengthened material of Hansen & Clauer (1973) at 400 °C ($0.72\,T_m$).

The following equation is taken for the jog concentration (Gittus 1974*a*):

$$c_j = \exp(-\mu b^3/8\pi kT). \tag{4}$$

Figure 2 shows how a dislocation can surmount a particle by climb-plus-glide. Climb occurs most readily at high temperatures and facilitates particle bypassing. Following Brown & Ham (1971) we can show that the following approximations are, theoretically, valid:

$$\left.\begin{aligned}\tfrac{1}{2}T_m \leqslant T < T_m &: x = 2T/T_m;\\ \tfrac{1}{2}T_m > T &: x = 1.\end{aligned}\right\} \tag{5}$$

Then using the values of the various parameters of equations (2), (3) and (4) collected together in Table 1 of Ashby's (1972*a*) paper on deformation maps we find that for $\sigma = 28.76$ MPa and $T = 673$ K with $b/xL = 1.36 \times 10^{-3}$, equation (2) gives a predicted steady-state creep rate of $5.8 \times 10^{-6}\,\mathrm{s}^{-1}$ while equation (3) gives the almost identical value of $4.1 \times 10^{-6}\,\mathrm{s}^{-1}$. We note in passing that at this temperature and stress the theory predicts the creep rate of the pure aluminium matrix with high accuracy. Comparison with Hansen & Clauer's data then shows that the 'theoretical material' has at 400 °C (673 K) the creep rate which the real material would exhibit at about 440 °C.

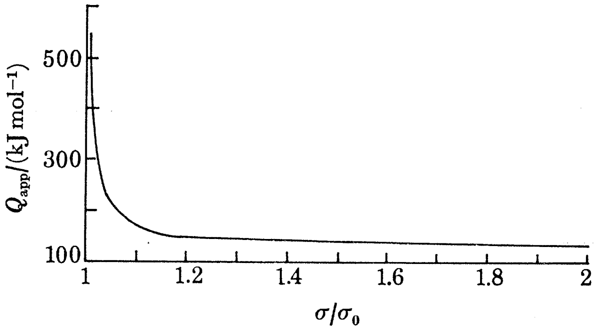


FIGURE 3. Theoretical effect of σ/σ_0 upon the apparent activation energy (Q_{app}) in the creep-equation (Al–$\mathrm{Al_2O_3}$ at 673 K).

Apparent activation energies

An empirical equation attributed to Nutting and Scott-Blair which is often used (Gittus 1964) to fit dislocation creep data has the following form:

$$\dot{\epsilon} = A\sigma^n \exp[-Q_{\mathrm{app}}/RT]. \tag{6}$$

In estimating the value of Q_{app} in equation (6) by the temperature-cycling method it is usual to assume that Q_{app} and n are both independent of temperature and stress so that we can calculate Q_{app} by differentiating equation (6) with both Q_{app} and n assumed constant:

$$Q_{\mathrm{app}} = -R\,\mathrm{d}\,(\ln\dot{\epsilon})/\mathrm{d}(T^{-1}). \tag{7}$$

Now, it is in fact incorrect, according to the theoretical equation for the creep of a dispersion strengthened alloy (equation (2)), to assume that Q_{app} and n are constants. If we differentiate equation (2) we can predict the value of Q_{app} which will be produced if the results of a temperature-cycling experiment are used to solve equation (6):

$$-Q_{\mathrm{app}} = -(Q_{\mathrm{v}}+Q_{\mathrm{j}}) + RT + 2RK_2\,T^2 - 3RT\frac{1-K_2\,T}{\sigma/\sigma_0-1}. \tag{8}$$

Here Q_{v} is the activation energy for volume self-diffusion and Q_{j} the activation energy for thermal jog-formation. Clearly the apparent activation energy, Q_{app}, depends on both the applied stress and on the mean temperature. Here

$$Q_{\mathrm{j}} = \mu R b^3/k8\pi \tag{9}$$

and

$$\mathrm{d}\mu/\mathrm{d}T = K_2\,\mu. \tag{10}$$

Using these equations together with data for aluminium taken from Ashby's (1972*a*) Table 1 for $T = 673$ K we find that $(Q_{\mathrm{v}}+Q_{\mathrm{j}}) = 130$ kJ/mol while at this temperature $K_2 = -5.4 \times 10^{-4}$.

From equation (5) we have $x = 1.44$ so we can calculate a value of Q_{app} for the dispersion-strengthened material of Hansen & Clauer (1973), using the parameter values of the previous section. Substituting these values into equation (8) we find that the apparent activation energy at 400 °C and a stress σ of 28.76 MPa should, theoretically, be 585 kJ/mol: this is the value which Hansen & Clauer calculated from their data for a stress of 27.68 MPa and temperatures in the range 400–500 °C. Figure 3 shows the general effect of stress on Q_{app}.

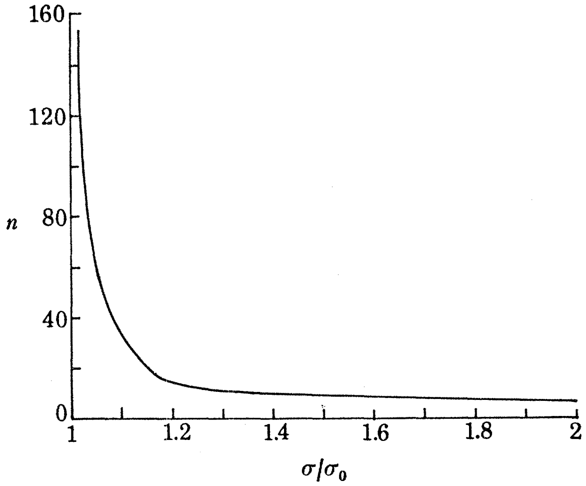


FIGURE 4. Theoretical effect of the ratio of applied stress to threshold-stress (σ/σ_0) upon the exponent of stress (n) in the creep-equation.

According to equation (8), the apparent activation energy should fall as the value of the externally applied stress is increased. This is what McLauchlin (1971) found in his studies of a stainless steel strengthened by a dispersion of NbC. McLauchlin's results also exhibit the high levels of apparent activation energy which the theory predicts. A high activation energy is found also in the results of Gulden & Shyne (1963) while Russell, Ham, Silcock & Willoughby (1968) found that in their experiments the apparent activation energy has the kind of stress dependence which our theory would have led us to expect. Similarly, Vickers & Greenfield (1968) report that in the case of Mg–MgO the apparent activation energy for steady-state creep is about 420 kJ/mol while that for self diffusion is only 135 kJ/mol. They review work on Al–Al_2O_3 which like the studies of Hansen & Clauer, yields activation energies (in the range 630–1260 kJ/mol) much higher than the self diffusion value.

Stress-sensitivities

Differentiating equation (6) with respect to stress we obtain

$$n = \mathrm{d}/(\ln \dot{\epsilon})/\mathrm{d}(\ln \sigma), \tag{11}$$

which with equation (2) gives

$$n = [3/(\sigma - \mu b/xL)]\,\sigma, \tag{12}$$

i.e.

$$n = 3/(1 - \sigma_0/\sigma), \tag{13}$$

and so

$$n \to \infty \quad \text{as} \quad \sigma \to \sigma_0; \quad n \to 3 \quad \text{as} \quad \sigma \to \infty. \tag{14}$$

Evidently the limiting behaviour summarized in relations (14) explains why, with increasing external stress, the value of n falls from the near-infinite values at low stresses towards an asymptotic value (see figure 4) ($n = 3$). A numerical example can be obtained by considering again the alumina dispersion strengthened aluminium at 400 °C and a stress of 28.76 MPa (the conditions which led, in the previous section, to a predicted activation energy of 585 kJ/mol). Then equation (12) gives $n = 78$. For comparison, Hansen & Clauer (1973) found that their data conformed to values of n ranging from 50 at 600 °C to over 100 at low stresses and 400 °C.

McLauchlin (1971) found that for his NbC-hardened stainless steel, the value of n was high and exhibited the kind of temperature dependence which equation (12) would predict. Crossland & Jones (1972) found that above 350 °C the stress exponent was of order 10 for Mg–MgO.

By substituting equation (13) into equation (8) we obtain

$$Q_{\text{app}} = Q_{\text{app},0} + 3RT(1-K_2T)(\tfrac{1}{3}n-1), \tag{15}$$

where $Q_{\text{app}} \to Q_{\text{app},0}$ as $n \to 3$. This illustrates the kind of relation between n and Q_{app} which was observed by McLauchlin (1971).

The conclusions which we have drawn about the values of the apparent activation energy and the exponent of stress for dispersion-strengthened alloys are supported by most of the relevant experimental data reviewed by Brown & Ham (1971). Support stems also from later work on a Nimonic alloy reported by Williams & Wilshire (1973).

In this section, then, we have shown how a tiny addition of a finely dispersed second phase can dramatically increase the deformation resistance of a pure metal or solid solution at elevated temperature. The effect is far greater than the equivalent effect of suspending particles in a liquid and it produces a creep rate whose sensitivity to changes in stress and temperature is not at all what one would expect from the behaviour of a pure metal. We shall now go on to examine what happens if the second phase, instead of being present in small amounts, is almost equal in volume to the first phase.

Superplasticity in two phase structures

The materials which exhibit what Johnson (1970) terms ‘fine-grain superplasticity’ may be single phase but those exhibiting the highest stability during deformation coupled with the largest extensions are more commonly multiphased. In the latter case it is frequently observed that the two phases are present in approximately equal proportions and that they appear to have approximately equal ductilities. Many of the superplastic alloys are therefore eutectics or eutectoids.

In these superplastic materials, individual grains or phase particles translate large distances relative to one another without exhibiting any large permanent changes in shape. This is illustrated in figure 5. In their studies of superplastic deformation in the Pb–Sn eutectic, Geckinli & Barrett (1976) used a special tensile stage in a scanning electron microscope. They photographed consecutive positions of the grains continuously during straining and it is from measurements made during these experiments that figure 5 was prepared. The grains rotated up to ± 30° from the tensile axis during the tests, often reversing the direction of their rotation during the course of straining. The grains remained equiaxed during straining although no voids were observed to form between them: evidently sufficient localized deformation was occurring to maintain coherency between adjacent grains as they slid past one another.

These observations caused Geckinli & Barrett to conclude that the dominant mode of deformation during superplastic flow is grain boundary sliding with localized fluctuating vibrations in grain shape. This view is consistent with the results of the majority of experiments in which superplastic flow has been produced (Gittus 1975 *b*).

There are several theories, but two have attracted the most interest. One of these, due to Ashby & Verrall (1973) proposes that a process akin to diffusion creep occurs. It allows the grains to make the fluctuating changes in shape that permit them to remain in contact during sliding. In effect all that need happen is for the material from some grain corners to diffuse to adjacent grain-faces, and vice versa. The interior of the grain does not alter in shape at all and may be regarded as a sphere which slides relative to the other spheres (grains). At low stresses and small grain sizes, Ashby & Verrall noted that the ability of the boundary to act as a source and sink for point defects might become limiting. The creep rate may then depend on the density of dislocations in the boundary. Now the Ashby & Verrall model, for the case in which the interface reaction is *not* rate controlling, leads to the prediction that the creep rate will depend on the first power of the applied stress, whereas experimentally it usually depends on the square or the cube. Interface-reaction control coupled with a sink-density which increases with stress, leads to a dependence of the strain-rate on the square of the stress.

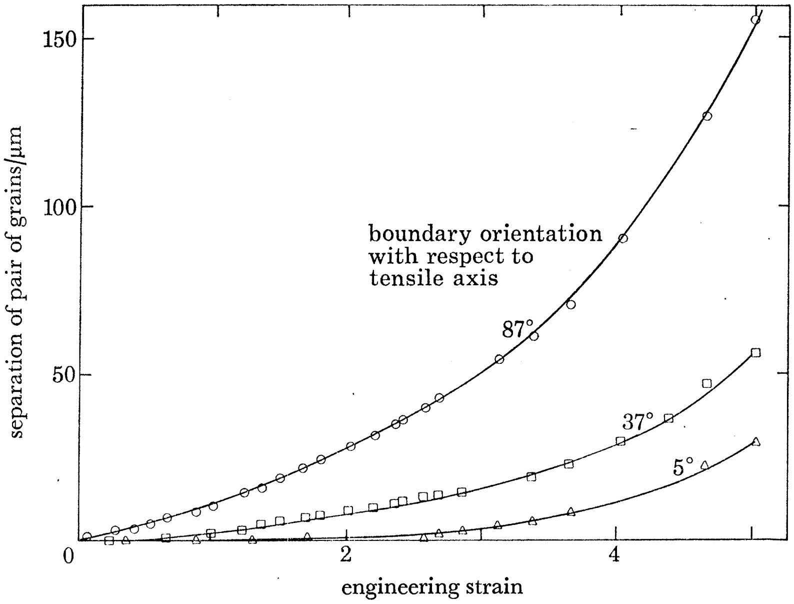


FIGURE 5. Separation of pairs of grains as a function of engineering strain. Pb–Sn eutectic. Reproduced by permission of Dr C. R. Barrett. Initial strain rate $5 \times 10^{-4}\,s^{-1}$; grain size 3.5 µm.

This kind of stress dependence was in fact a characteristic of an earlier model of superplastic flow put forward by Ball & Hutchison (1969). Their model, as they remark, is virtually the same as one suggested earlier by Friedel (1964) to explain the fact that the activation energy for the creep of fine grained zinc is that for grain-boundary diffusion. They suggested that at high temperatures dislocations from the head of a pileup would climb into the boundary against which they were piled up. Sources on the slip plane, by operating to replace the dislocations so 'lost', produced creep. They were led to a creep equation having the observed type of dependence of strain-rate upon both stress and grain size. Mohamed, Shei & Langdon (1975) have examined the Ball & Hutchison model, particularly in relation to superplasticity in the Zn–Al eutectic and have pointed to two drawbacks: first, the model assumes that adjacent grains slide in groups and this is not supported by direct observations in the scanning electron microscope; secondly, the model predicts the presence of many intergranular dislocations although transmission electron microscopy and internal marker experiments have been interpreted to indicate that there is very little within-grain dislocation activity during superplastic flow. Another problem is that superplastic flow is sometimes observed to cease at a low stress and neither theory can account for the magnitude of this 'threshold' stress.

Neither of the two theories then is entirely satisfactory, even for the case of superplastic flow in a single phase alloy. In the case of a two phase alloy there are additional difficulties due to the nature of the sliding and climb processes at interphase boundaries. We consider this matter in the next section.

Sliding, growth and dissolution at interphase boundaries

It is, of course, established that particles of a second phase can precipitate and dissolve. In addition processes such as 'ripening' are observed in which material from one particle of a second phase diffuses to another through the intervening matrix. Again, the shape of a particle can change, presumably by the diffusion of matter from one region to another. These mechanisms are capable then of producing the changes in particle shape (and possibly, size) which have to occur in order to preserve continuity at the contacting faces of particles and matrix during superplastic flow. Aaronson, Laird & Kinsman (1970) have summarized the mechanisms of these diffusional growth processes for precipitate crystals. The orientation relation of the precipitate lattice with respect to the matrix lattice is assumed fixed during nucleation. The structure of the interphase boundary varies with the orientation of the boundary. Dislocation structures appear at one or two orientations and disordered structures at all of the others. In Aaronson's original theory it had been assumed that both the dislocation boundaries and the disordered boundaries had structures which closely approximated their equilibrium configurations. At least in the case of dislocation boundaries it has since become clear that the details of the misfit dislocation structure are often greatly dependent upon the mechanics and the kinetics of generation or introduction of misfit dislocations. The dislocation boundaries can only migrate by the movement along their length of ledges. The narrow faces of these ledges often appear to be disordered and so there is no structural barrier to precipitation of additional material onto them. Similarly there is no structural barrier to the growth (migration) of the disordered boundaries. In the ordered boundaries on the other hand, addition of a new layer of atoms will generally alter the packing relation between the two contacting lattices, introducing a measure of disorder. The energy needed to accomplish this is usually high. It is for this reason that precipitates are often in the form of long plates or rods: the long dimension is a dislocation boundary which can only thicken by the slow process of ledge-growth whereas elongation of the precipitate occurs preferentially by the addition of atoms to regions of the boundary that are already so disordered that the new atoms cannot raise their energy significantly.

Sliding at dislocation interphase boundaries is impeded by the same type of energy barrier. Thus the passage of a glissile dislocation leaves in its wake a disordered structure of high energy. The energy increase could be so high as to prohibit the glide and with it any sliding at the interphase boundary. That is to say the threshold stress for i.p.b. sliding could be very high (Ashby 1972*b*). Dislocation boundaries in single phase materials can slide more easily. They do this, in most cases, by a combination of translation and migration. Atoms are transferred across the interface during sliding to restore the boundary structure, which would otherwise be altered by the sliding process. In this way one of the grains grows at the expense of the other. If atoms are transferred across the interface between two different phases, then the grain which grows does so by acquiring the wrong kind of atoms – those which constituted the other phase. Equilibrium can be restored by the diffusional redistribution of the components but this introduces a large viscosity term since the diffusion path is of the order of the grain size.

It is now suggested, however, that there *is* a mechanism of sliding available on dislocation interphase boundaries. It involves the glide of a superdislocation in the boundary plane. Figure 6 shows what happens. The first of a pair of boundary dislocations creates disorder but the second dislocation, when it passes along the same path, restores order once more. Nakagawa & Weatherly (1972) have actually observed pairs of dislocations in the interface between two ordered lattices. The lattices were those of Ni_3Al and Ni_3Nb in a directionally solidified eutectic alloy. The dislocation pairs had separations of 30–40 nm. They were epitaxial dislocations with $b = a/2[1\bar{1}0]_{\mathrm{Ni_3Al}}$ and the orientation relation between the phases at their common boundary was $(1\bar{1}0)_{\mathrm{Ni_3Al}} \| (100)_{\mathrm{Ni_3Nb}}$, $(113)_{\mathrm{Ni_3Al}} \| (031)_{\mathrm{Ni_3Nb}}$. What we are now suggesting is that the formation of superdislocations during deformation may be a quite general phenomenon in interphase boundaries, even where the contacting phases do not themselves exhibit long-range order. An interphase boundary between two pure substances (α and β) is in fact an ordered solid solution of one phase in the other and the disordered region between the leading and the trailing components of the superdislocation is an antiphase boundary (a.p.b.). The Burgers vector can make any angle to the i.p.b. In figure 6, ***b*** lies in the i.p.b. If ***b*** is perpendicular to the i.p.b. then the a.p.b. is equivalent in structure to a faulted dislocation-loop. The superdislocation now moves by climb, vacancies diffusing through the fault to the trailing edge from the leading edge. More generally a mixture of climb and glide will occur, the latter leading to i.p.b.-sliding.

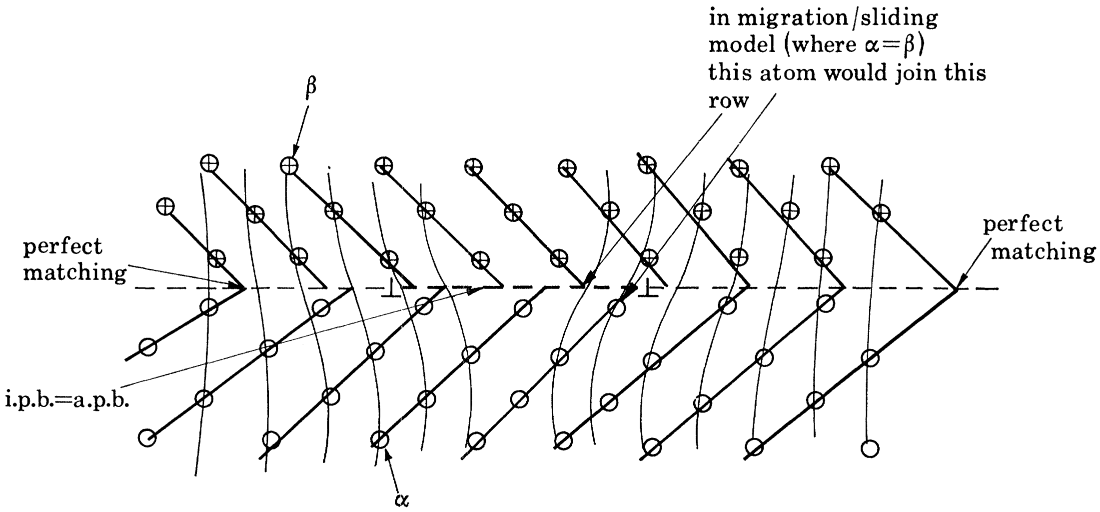


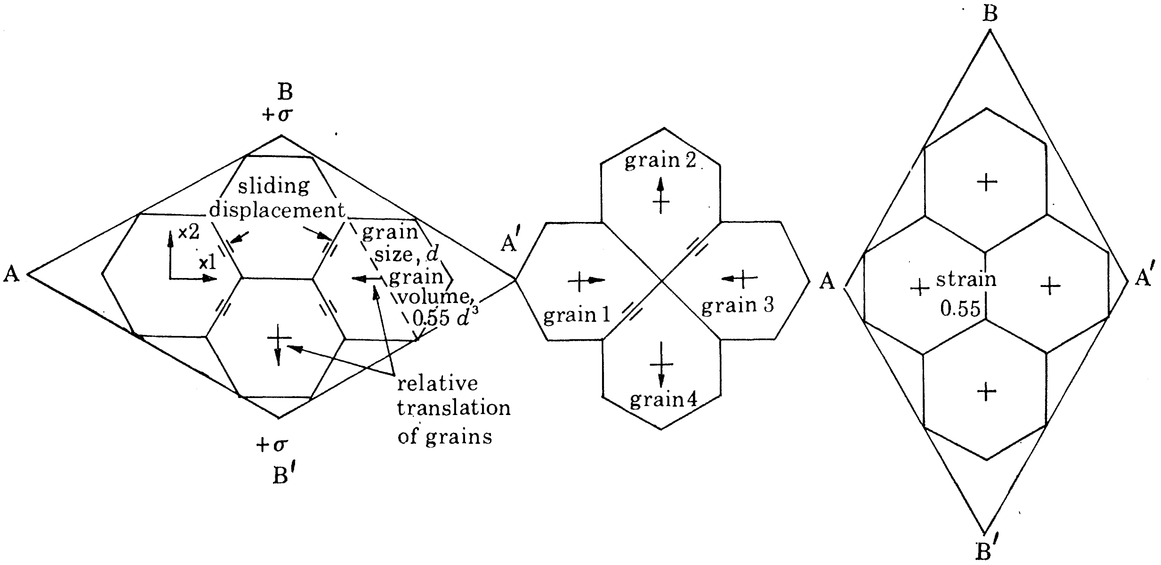
FIGURE 6. Interphase boundary (i.p.b.: – – –) sliding by motion of an i.p.b.–superdislocation contrasted with the Ashby migration/sliding process. The latter involves crystallization of α-atoms on the β-lattice or their diffusion along paths comparable in length to the grain-size: this produces a high i.p.b. viscosity. The viscosity is characteristically much lower if an i.p.b. superdislocation glides in the boundary. The two partial dislocations are then separated by a region of i.p.b. having the structure of an antiphase domain boundary (a.p.b.: ---).

The equilibrium width (w) of the disordered region can be calculated from a knowledge of the elastic constants and a fault energy, Γ (Gittus 1977*a*). Thus for the case where superdislocations occur with separation l, we can define a long-range order parameter, S, as

$$S = 1 - w/l, \tag{16}$$

so that, when the dislocations are so close together that the partials touch, $S = 0$ corresponding to the complete absence of long range order (a disordered boundary). The width of the superdislocation is then given, to the first approximation, by an equation essentially similar to that (Haasen 1965) for the width of the superdislocations in an ordered solid solution:

$$w = \frac{\mu b^2}{2\pi \Gamma S^2}. \tag{17}$$



(*a*) initial state (*b*) intermediate state (*c*) final state

FIGURE 7. Unit step in the deformation process observed by Naziri *et al.* (1973) and modelled by Ashby & Verrall (1973). The lozenge A B A′ B′ undergoes a shear of magnitude 0.55 by the moving together of A and A′ and the simultaneous moving apart of B and B′.

As an example consider the case where the energy of the disordered boundary in the fault, Γ, is 100 erg cm^{-2} and let $b = 2 \times 10^{-8}$ cm, $\mu = 5 \times 10^{11}$ dyn cm^{-2}, and $S = 0.75$. Then

$$w = 5\,\text{nm}.$$

The disordered region is identical in structure with the narrow face of a growth ledge and we may in fact regard such ledges as superdislocations in which the fault and glide-planes are no longer coparallel. Such 'staggered' superdislocations are certainly observed in grain-interiors.

A new theory for superplastic flow in two phase materials

In the previous section we have argued that particles of differing phases can slide relative to one another by the movement of interfacial superdislocations. We have also remarked that a particle can change its shape or size by the diffusion of matter to and from disordered regions of its boundary. These principles have now been incorporated (Gittus 1977*a*) in a model of the superplastic flow process for two phase materials. That model is as follows:

The unit process is taken to be the grain-switching event observed during the superplastic flow of the Zn–Al eutectoid by Naziri, Pearce, Henderson-Brown & Hale (1973). It is the unit process which Ashby & Verrall analysed (1973) and is illustrated in figures 7–9. Figure 7 shows what Naziri *et al.* observed in the high-voltage microscope. Grains of the zinc-rich phase (labelled 1 and 3 in figure 7*b*) push into the boundary between two grains of the Al-rich phase, separating the latter. The aggregate of four grains undergoes a shear of magnitude 0.55 in this process but the final shape of each grain is the same as its starting shape. In fact (figures 7 and 8) each face of a grain becomes a corner and each corner a face. Figure 8 also shows the length of the diffusive paths, λ, along which matter flows to accomplish the shape-changes. It is on average 0.3 d. By contrast, in Herring–Nabarro diffusion creep the mean diffusion path is approximately equal to

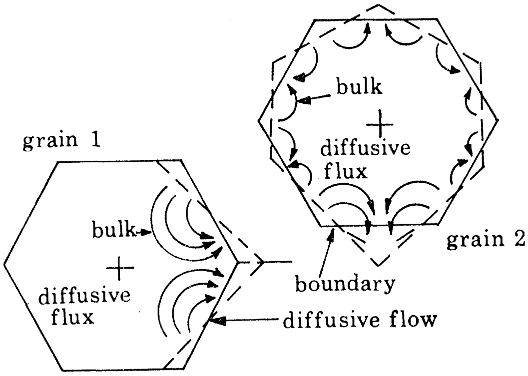


Figure 8. In order to maintain contact at the interphase boundaries during the grain switching process of figure 7, material must diffuse between the points and facets of the grains. This flow occurs over short distances and only enough material need be transported to prevent cavitation. After Ashby & Verrall (1973).

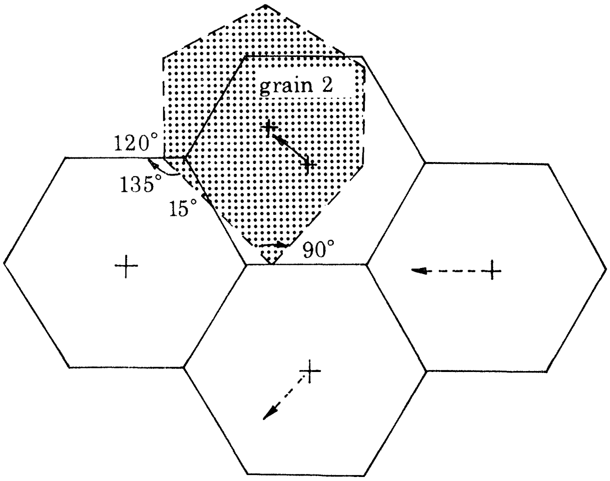


Figure 9. Here, superimposed on the initial state (figure 7a), is shown the intermediate position of grain 2. Matter has diffused to produce a new apex (point). In addition the centre of the grain has translated and a shear displacement equal to the amount of this translation has occurred at the interphase boundary. After Ashby & Verrall (1973).

half the grain diameter. Again only about one sixth of the volume of the grain has to diffuse during the unit process in order to produce unit of strain. In the Herring–Nabarro process on the other hand the whole volume of a grain must suffer diffusion transport to produce unit of strain. Figure 9 shows that the grains slide relative to one another during the unit process. It is these shear displacements in the boundary that permit the grains to translate past each other. Their magnitude can be seen by examining the motion of grains relative to a fixed origin, say the centre of mass of grain 2 in figure 9.

Suppose that i.p.b. superdislocations can be generated by (for example) the expansion of intrinsic dislocation loops under the applied stress. In the boundary the dislocations are effectively trapped and can only emerge into the matrix if they reassociate to form matrix dislocations of appropriate Burgers vector. The dislocation boundary is therefore potentially a very effective pileup of length d. The movement of dislocations in this dynamic pileup causes i.p.b. sliding and consequently creep. It proceeds at a rate determined by the climb of the dislocations away from the head of the pileup. They cannot generally climb out of the boundary; therefore they must climb in it. The most likely situation is one in which the dislocations pile up at a triple point where a dislocation boundary on which they are gliding meets two disordered boundaries into which the dislocations then climb. For this climb to occur matter must diffuse to or from the dislocations (diffusion coefficient = D_B, diffusion distance λ).

Analysis of this model (Gittus 1977*a*) finally yields the *constitutive equation for superplastic-flow*:

$$\dot{\epsilon} = 4.8\left(\frac{d}{\lambda}\right)^2 \frac{D_B \mu b}{kT}\left(\frac{\sigma-\sigma_0'}{\mu}\right)^2\left(\frac{b}{d}\right)^2, \tag{18}$$

where d is the particle size, D_B the i.p.b. diffusion-coefficient and σ_0' a threshold stress. In equation (18) the strain rate is independent of particle size except through the effect of d on the diffusion distance λ: on most models of the flow process, λ = constant × d. The constant is of magnitude 0.3 in the Ashby–Verrall grain-switching model.

Structural effects

The dislocation-spacing on dislocation-boundaries theoretically decreases with increasing stress. Hence (equation (16)) the long-range order parameter, S, will fall as the stress increases. Now a completely disordered boundary ($S = 0$) has no preferred relation to the lattices of the contacting phases and tends to adopt a spherical shape to minimize its area (and thus to minimize the surface energy). At the other extreme a dislocation-boundary ($S \to 1$) follows preferred crystallographic directions and is essentially planar. When a two phase material is stressed, then, we expect the gradual build-up of interphase boundary superdislocations towards the equilibrium density (and the associated fall in S) to make more and more of the interphase boundary adopt a curved shape. This is observed (Gittus 1975*b*) and in the earlier literature led to the idea that the formation of 'bulbous' structures was a prerequisite of superplasticity.

Source of the threshold stress, σ_0'

It is suggested (Gittus 1977*a*) that the threshold stress, σ_0', is due to an interaction between i.p.b. superdislocations and boundary-ledges. To traverse a ledged boundary the dislocations will have to bend. Moreover there will be a change in fault energy of magnitude Γwh per intersection, where h is the height of a ledge, when the narrow face of a ledge is a disordered i.p.b. The applied stress will have to do work in order to overcome this energy in order to detach a dislocation from a ledge. Garmong & Rhodes (1975) saw evidence of these interactions between i.p.b. dislocations and ledges in their transmission electron microscope studies of Ni_3Al–Ni_3Nb composites. The interaction was manifested both as a local alteration of the line vector of the dislocation in certain circumstances and as a change in the reponse of the dislocation image in image-contrast experiments.

A suggested model of the ledge-dislocation network is a two dimensional dislocation network. In order to detach itself from two ledges a boundary dislocation may be forced to adopt a curvature C as high as $2/L$, where L is the width of the broad face of a ledge (i.e. the distance between adjacent narrow faces). If the boundary defect is regarded as a flexible line whose energy per unit length is E, a potential

$$\Delta\mu = E\Omega C/b \approx 2E\Omega/bl \tag{19}$$

is required to do this (Bardeen & Herring 1952; Thompson & Balluffi 1962). The threshold stress, σ_0', can supply this potential when $\sigma_0'\Omega \geqslant \Delta\mu$. Here Ω is the atomic volume. This, together with equation (19), then gives as an estimate of the threshold stress due to the interaction of glissile i.p.b. superdislocations and ledges:

$$\sigma_0 \leqslant 2E/bL. \tag{20}$$

Precipitates lying in the boundary are another source of a threshold stress. An equation similar to (20) then applies but with L equal to the spacing between the precipitate particles. Solute-drag may also tend to restrict the movement of the boundary super-dislocations. However, this will reduce the rate of glide at all stresses, rather than introduce a threshold stress (Gittus 1974*b*).

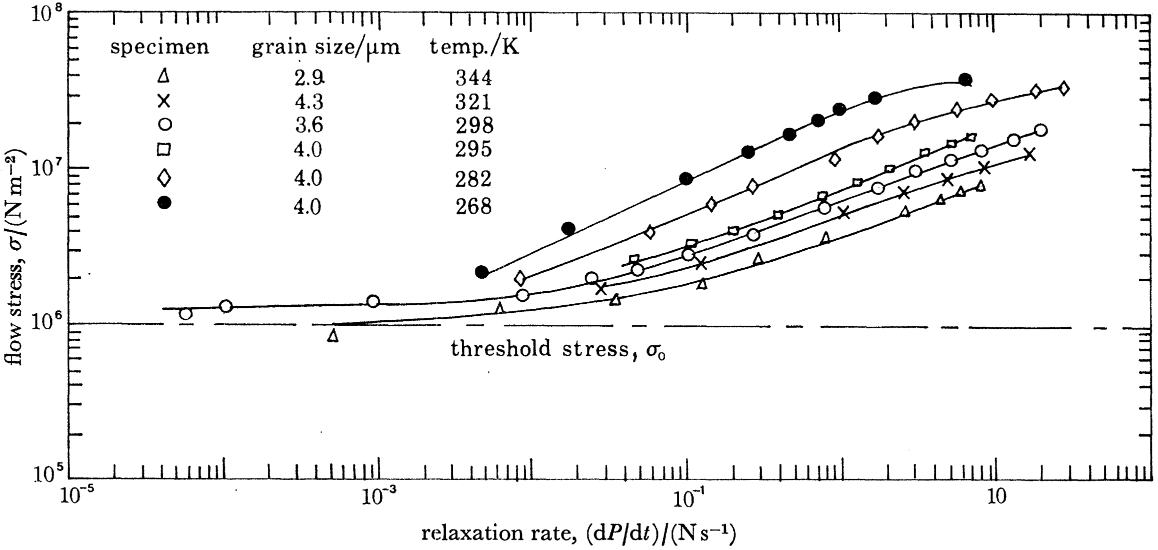


FIGURE 10. Illustrating the 'sigma-shape' of the curve relating log strain rate to log stress. Superplastic Pb–Sn eutectic. The relaxation rate is the rate at which the load fell in a stress relaxation test and must be multiplied by a factor to arrive at a strain rate. In other plots no effect of grain size upon the value of the threshold stress was found. Data of Geckinli & Barrett (1976).

Comparison with the results of experiment

Detailed comparison has been made (Gittus 1977*a*) between equation (18), the theoretical relation for superplastic flow, and the published results of experiments. Good agreement is generally found and the value of λ ($0.3\,d$) demanded by the grain-switching model is often closely approximated. For example Misro & Mukherjee (1974) found that equation (18) fitted their data for the Al–Zn eutectoid with a constant of proportionality which in the present terminology implies that $\lambda = 0.19\,d$: this is close to the value of $0.33\,d$ which best fits Ball & Hutchison's data (1969) for the same material. Again Alden has reported on the superplastic flow of a Sn–5 % Bi alloy (1967) and in this case the best value is $\lambda = 0.45\,d$.

Magnitude of the threshold stress

A very clear illustration of the threshold effect is contained in figure 10 (Geckinli & Barrett 1976). In this work the specimens were first deformed in tension until the maximum flow stress was attained. The cross head motion was then stopped and unloading characteristics recorded. The threshold stress is about 1.3 MPa at room temperature. One possibility, which Geckinli & Barrett consider, is that the threshold is determined by the work which has to be done to produce the changes in boundary area which accompany the fluctuating changes in the shape of the *particles*. They show that this would yield a threshold stress about an order of magnitude smaller than the one they measured and Ashby & Verrall (1973) who proposed this source of a threshold stress confirm that this is a typical discrepancy. If we are correct in ascribing the threshold stress to a ledge-dislocation interaction, then (equation (20)) in Geckinli & Barrett's work the value of L is calculated to have been of the order of 400 nm. This is of the order of magnitude observed on dislocation-boundaries. That the ledges do impede the movement of boundary dislocations is established by the electron microscopy reported by Garmong & Rhodes (1975). These workers calculated that in their Ni_3Al–Ni_3Nb eutectic composites the pinning stress acting to pin the i.p.b. dislocations to the ledges would, theoretically, be $\mu/70$. The threshold stress for dislocation movement is expected to be about h/L times this value, for the applied stress acts on the entire unpinned length of dislocation (minimum length L) while the pinning stress only acts on the length of dislocation near to the ledge (height h). So

$$\sigma_0' \approx \mu h/70L.$$

If this equation, derived for a different material, nevertheless holds good for the Pb–Sn eutectic then the observed value of the threshold stress in that material would imply that the ledges on the dislocation boundaries are about 100 times as long as they are high. This is in reasonable accord with experiment.

Activation energies

A change in temperature can alter the proportions of the phases in a superplastic alloy. As the new structure will not have the same strain rate–stress relation as the old one, the apparent activation energy determined by a temperature-change experiment can include a spurious effect due to the structural change. Of work in which an attempt has been made to avoid this latter pitfall, that of Herriot, Baudelet & Jonas (1977) and of Suery & Baudelet (1975) is of special interest. These workers found in their studies of the two phase Cu–P alloys that the activation energy for superplastic flow, calculated with no endeavour to correct for change in structure, was close to that for grain boundary diffusion in copper. This is the kind of result that appears to fit in quite well with the present theory. When, however, allowance was made for changes in *structure* the activation energy for flow rose to a value quite close to that for volume self diffusion in copper. On the face of it this would seem then to run contrary to our theory. However, Perinet (1975) and Bondy, Regnier & Levy (1971) have found that a high value of the activation energy is characteristic of impurity diffusion at interphase (as opposed to grain-) boundaries. Thus Perinet found that the activation energy for diffusion in the boundary between copper and silver was 65 kcal mol^{-1} and this is more than twice the grain boundary activation energy for either of the pure elements by itself. Again a similar finding was reported for the diffusion of Fe and Ag in the interface between these two metals. Experiments to measure self-diffusion in i.p.bs have been planned by Chadwick & Gittus (1977) and are now in progress.

Initial results show that self-diffusion at an i.p.b. has much in common with impurity-diffusion, being a spectacularly rapid process with an activation energy nearer to that for volume diffusion than to that for grain boundary diffusion. These findings clearly accord, on the present theory, with the high value of activation energy for superplastic flow at constant structure found by Baudelet and his collaborators. Indeed this may be a general feature of superplastic flow in two phase materials, having passed unnoticed in earlier investigations because care has not always been taken to compensate for the effect of changing structure when calculating the activation energy for superplastic flow. When Baudelet omitted this precaution he obtained a spuriously low activation energy which is, coincidentally it now appears, similar to that for grain boundary diffusion (as opposed to phase boundary diffusion) in one of the pure components of which his Cu–P alloy was composed.

The transition to non-superplastic deformation

If the strain rate is increased sufficiently, dislocations start to appear in considerable numbers inside the grains or particles of a superplastic material. It loses its high ductility and the strain rate varies now with a higher power of the externally applied stress. 'Conventional' dislocation creep has begun. In recent work a method of deducing the critical stress at which this change in deformation character will occur has been developed (Gittus 1977*b*). Details are given below.

Thermodynamic considerations

Consider a specimen undergoing dislocation creep at a constant strain-rate, $\dot{\epsilon}$, above that at which superplasticity is observed. Characteristically it will, at the outset of the creep process, contain a random three dimensional dislocation network. With the passage of time the dislocations form themselves into cells. Typically the cell diameter and dislocation density both move towards an equilibrium value during this initial period (McElroy & Szkopiak 1972). The large extensions which can occur during this steady state strongly suggest that it is a condition of thermodynamic equilibrium, characterized by a minimum free energy.

At this thermodynamic equilibrium:

$$(\delta F)_{T,V} \geqslant 0. \tag{21}$$

Here, after Shewmon (1965), the lower case delta (δ) is used to indicate that the free energy is unchanged or increased for any small (finite) displacement of the system.

Components of the free energy of the specimen

The dislocations which are contained in the creep specimen make an important contribution to its free energy. That contribution depends on the dislocation density and upon the arrangement of the dislocations characterized by a quantity K. If $K = 1$ then the dislocations are arranged in a uniform three dimensional network. If $K > 1$ then the dislocation arrangement consists of cells having most of the dislocations in their walls or boundaries. The larger is K, the bigger is the cell diameter (for a given dislocation density).

Another component of the free energy of the specimen is the elastic plus viscoelastic strain-energy produced by the externally appplied stress. If this were the only component, then the lower the applied stress the lower the free energy. So at a fixed strain rate the dislocations would tend to move into that arrangement which minimizes the stress that the external machine has to exert. The condition of thermodynamic equilibrium could then be the condition of minimum creep-strength. We can show that there is a certain value of K (of order $c_j^{-\frac{1}{3}}$) which produces this minimum creep strength. If we now consider both components of the free energy (that due to dislocations and that due to elastic-plus-viscoelastic strain) then we can calculate a more exact estimate of the value of K which minimizes the free energy. It transpires (Gittus 1977*b*) that

$$\frac{\mathrm{d}F}{\mathrm{d}K} = \frac{-1}{\mu}\left(\frac{\dot{\epsilon}}{B}\right)^{\frac{2}{3}} K^{-3} + \frac{1}{\mu}\left(\frac{\dot{\epsilon}}{B}\right)^{\frac{2}{3}} \left(-2K^{-3} + \tfrac{1}{5}c_j^2K^3 + \tfrac{3}{10}C'c_j^2K^5\right)\left(K^{-2} + \tfrac{1}{20}c_j^2K^4\right). \tag{22}$$

Here

$$B = c_j\,D_v\,b/(\mu^2kT), \tag{23}$$

$$C' = ((\mu/\mu_R) - 1)/K^2, \tag{24}$$

and μ_R is the relaxed modulus.

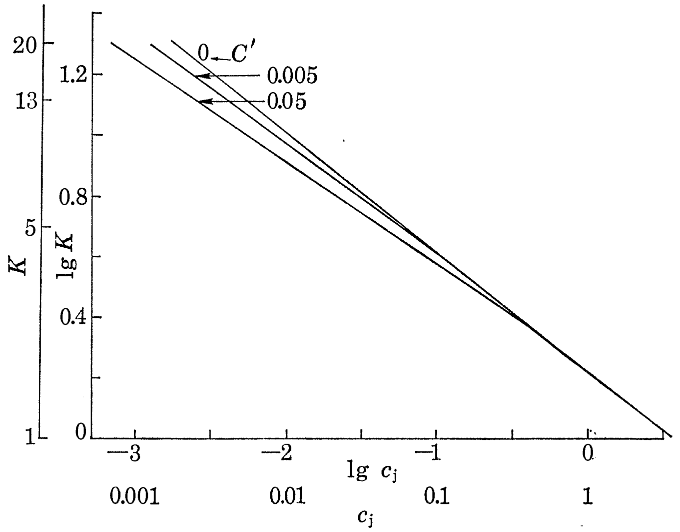


FIGURE 11. Theoretical value of K as a function of c_j for various values of C'. Here $C'K^2 = (\mu - \mu_R)/\mu_R$ where μ and μ_R are the shear modulus and the relaxed modulus respectively. Theoretically $0 \leqslant C' \leqslant 0.05$. In practice its value is usually less than 0.005.

The condition set by equation (21) is met if $\mathrm{d}F/\mathrm{d}K = 0$ and $\mathrm{d}^2F/\mathrm{d}K^2 =$ positive. Equation (22) with $\mathrm{d}F/\mathrm{d}K = 0$ gives

$$c_j^2 = \{-b + (b^2 - 4ac)^{\frac{1}{2}}\}/2a, \tag{25}$$

where

$$\left.\begin{aligned} a &= 10^{-2}K^7 + \tfrac{3}{200}C'K^9; \\ b &= \tfrac{3}{10}C'K^3 + \tfrac{1}{10}K; \\ c &= -K^{-3} - 2K^{-5}. \end{aligned}\right\} \tag{26}$$

Differentiating equation (22) a second time we find that the condition $\mathrm{d}^2F/\mathrm{d}K^2 =$ positive is met. So equations (25) and (26) correspond to a state of thermodynamic equilibrium. From them the theoretical value of K may be calculated.

Theoretical relation between K, C' and c_j

In figure 11 equation (25) has been used to provide the theoretical relation between K and c_j for $C' = 0$, 0.005 and 0.05. $C' = 0.05$ is the maximum theoretical value of this parameter, corresponding to the case in which all of the dislocations in the cell walls bow out reversibly under stress, contributing to the viscoelastic strain (Friedel 1964). In practice the measured ratios of relaxed to unrelaxed modulus correspond to C'-values of order 0.005 (i.e. only about one tenth of the dislocations bow in this manner).

For materials in use at the temperatures where dislocation-creep is the dominant deformation mechanism we generally have $0.1 > c_j > 0.001$ and so theoretically $20 > K > 5$: the values of K that have been measured experimentally do in fact lie generally in this range (Gittus 1975*b*).

TABLE 1. THEORETICAL RELATION BETWEEN THE VALUE OF K AND THOSE OF c_j AND K' FOR VARIOUS VALUES OF C'

C'	K	c_j	c_j (approx.)	K'
0	1	3.61	3.16	1.65
	5	5.50×10^{-2}	5.62×10^{-2}	3.36
	13	5.11×10^{-3}	5.13×10^{-3}	7.30
	20	1.75×10^{-3}	1.74×10^{-3}	10.80
0.001	10	9.42×10^{-3}	9.04×10^{-3}	5.44
0.005	1	3.60	9.12	1.65
	5	5.20×10^{-2}	7.24×10^{-2}	3.11
	13	3.80×10^{-3}	4.17×10^{-3}	4.48
	20	1.25×10^{-3}	1.15×10^{-3}	6.60
0.05	1	3.49	5.13	1.61
	5	3.87×10^{-2}	4.07×10^{-2}	2.17
	13	2.37×10^{-3}	2.34×10^{-3}	2.36
	20	6.87×10^{-4}	6.46×10^{-4}	2.51

Note: if $C' = 0$ then $K \approx 1.58\,c_j^{-0.4}$; $K' = 1 + K/2$.

Approximations to K

For finite values of C' and for $K > 5$, equation (25) reduces to the following approximation:

$$K \approx [0.03\,C']^{-\frac{1}{12}}\,c_j^{-\frac{1}{3}}. \tag{27}$$

For $C' = 0$ and for $K \geqslant 1$ the following alternative approximation to equation (25) is valid:

$$K \approx 1.58\,c_j^{-0.4}. \tag{28}$$

To illustrate the range of validity of these approximations, equations (27) and (28) have been used to produce estimates of c_j (c_j (approx.)) for selected values of K and C'. The results are compared in table 1 with the exact values of c_j derived from the full solution of equation (25). Only in the case of $K = 1$ for $C' = 0.005$ and 0.05 are the approximations poor: for most purposes, the approximations are acceptable and lead to some simplification of the creep equation.

Theoretical relation between cell diameter and stress

It has been shown (Gittus 1976) that, according to the present theory, the following equation should relate the stress causing creep to the diameter of the cells, L':

$$\sigma = (\mu b/L')\,(1 + \tfrac{1}{20}c_j^2\,K^6). \tag{29}$$

Experiments indicate (Staker & Holt 1972) that

$$\sigma \approx K'\mu b/L', \tag{30}$$

with

$$K' \approx 10. \tag{31}$$

A theoretical value of K' can be obtained by solving equation (29) for K' using the values of K and c_j from table 1. This has been done and the theoretical values of K' so obtained are there tabulated. The theoretical K' values are seen to be of the expected magnitude.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Deformation maps for the transition

We can now begin to see what happens as the stress is reduced in a two phase material undergoing dislocation creep. In each of the particles of the two phases there is a network of dislocations arranged in cells. As the stress is reduced, the equilibrium cell diameter, L, increases (equation (29) and table 1 with equation (30)). We can see that at a low enough value of the stress the cell diameter will become equal to the particle size and at that point the whole of the strain must be accounted for by superplastic flow. At some higher stress the strain due to superplastic deformation mechanisms will be equal to that due to dislocation creep. This will occur when the *cell* boundary area is about equal to the *interphase* area, or more precisely when

$$L' \approx d/3. \tag{32}$$

So from equations (30) and (32):

$$d/b = 3K' (\sigma/\mu)^{-1}. \tag{33}$$

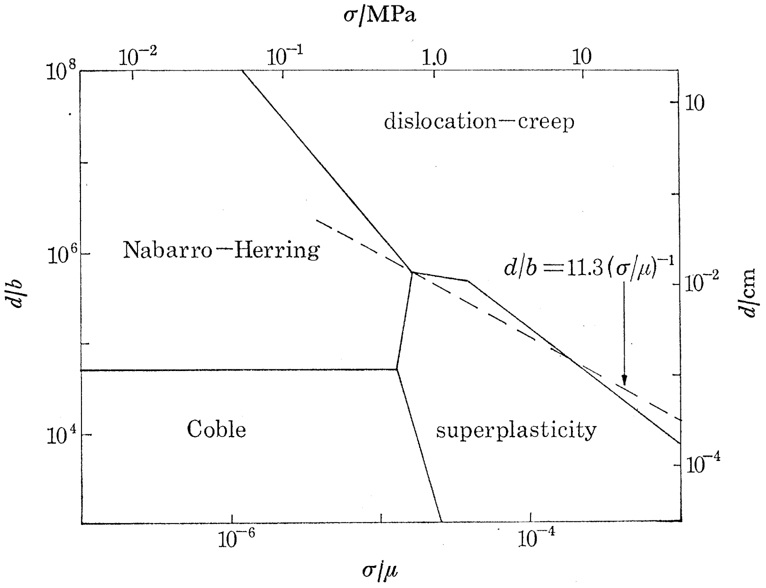


FIGURE 12. Deformation mechanism map prepared by Mohamed & Langdon (1976) for Zn–22 % Al. Theoretical line for the boundary between superplasticity and dislocation-creep.

The validity of equation (33) can be assessed by means of deformation maps prepared by Mohamed & Langdon (1976) for Zn–22 % Al and Pb–62 % Sn. These are reproduced here as figures 12 and 13 and on them have been drawn the theoretical lines corresponding to equation (33). The appropriate values of K' were calculated using table 1 and were as follows:

$$\text{Zn–22 \% Al,} \quad K' = 3.78; \tag{34}$$

$$\text{Pb–62 \% Sn,} \quad K' = 2.31. \tag{35}$$

The theoretical lines according to equation (33) with the values given in equations (34) and (35) have been plotted in figures 12 and 13 and are seen to give a good indication of the boundary where superplastic deformation produces the same contribution to the flow rate as does dislocation creep.

Source of the high ductility in superplastic flow

It has long been difficult to explain just why the ductility in the superplastic régime should be so high. Certainly if $m = 1$ then one would expect quasi-infinite ductility (Newtonian viscous flow) since it is easy to show that in such a case a 'neck' will not propagate. A neck is a local region of reduced diameter in a tensile specimen. It forms as a result of some local inhomogeneity of structure or temperature. If thinning occurs faster at the neck than elsewhere, the specimen will eventually neck-down to a point and fracture. This is what happens in ordinary creep. It does not happen (and is not expected) if $m = 1$. The anomaly has been that necks do not propagate in superplastic materials where m can be as low as 0.3 and is theoretically (we have shown) equal to 0.5.

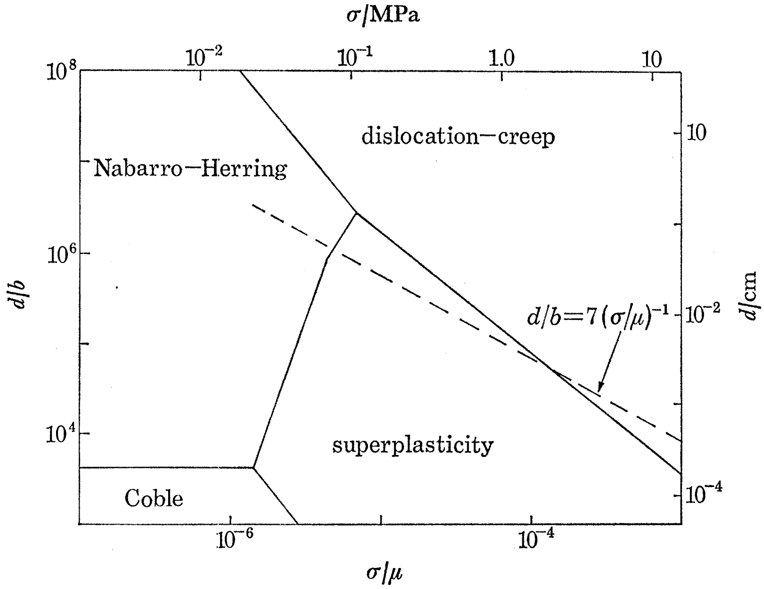


FIGURE 13. Deformation mechanism map prepared by Mohamed & Langdon (1976) for Pb–62% Sn. Theoretical line for the boundary between superplasticity and dislocation-creep.

An explanation can now be offered. It is simply that when the cell-diameter becomes equal to the particle (grain) size *there is a further reduction in the free energy of the specimen.* This is partly due to the low Burgers vector values of i.p.b. dislocations and partly because the dislocation-pileups on the i.p.bs, by concentrating the stress causing climb, reduce the stress needed to cause a given strain rate. We have shown that cells adopt a size which minimizes free energy during ordinary dislocation creep. When that size is equal to the particle (grain) size there is a further step-reduction in free energy. If a neck does try to propagate in the superplastic state then the associated local increase in stress, by reducing the equilibrium cell size below the particle size, produces a large increase in the free energy of the system. So neck propagation is strongly resisted.

DISLOCATION CREEP

Equation (29) is an approximation, taken from a more exact analysis of dislocation creep (Gittus 1976) which has led to the following equation:

$$\sigma = \sigma_{\perp} + F', \tag{36}$$

where
$$\sigma_\perp = (\dot{\epsilon}/B)^{\frac{1}{3}}/K^2; \tag{37}$$
$$F' = (kT/b^2l')\,\mathrm{arsinh}\,(\sigma_\perp b^3 c_\mathrm{j} K^6/20\,kT). \tag{38}$$

Here l' is the jog-separation. The equation agrees quantitatively with creep data.

The sensitivity of the creep rate to a change in the externally applied stress, predicted by these equations, falls as the stress moves towards zero (figure 14). So (from table 2) $n = \mathrm{d}\lg\dot{\epsilon}/\mathrm{d}\lg\sigma$ for gold (at a temperature T equal to half the absolute temperature of melting) is predicted to fall from a value of about 5 toward its low-stress asymptote of 3 as the applied stress falls from 10^{-3} to $10^{-4}\mu$. As K (the ratio of the cell size to the dislocation-spacing) becomes large the equations reduce to one found (by Garofalo) to give a good fit to creep data over an extended range of temperatures and stresses. Most of the temperature dependence arises from that of the volume self-diffusion coefficient, D_v; again this is in general accord with the results of experiment.

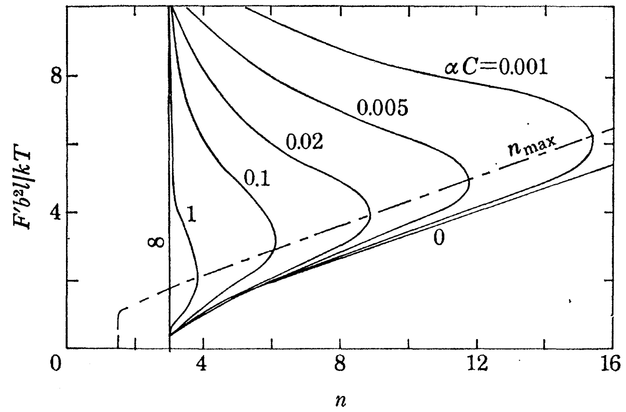


FIGURE 14. Relation between $F'b^2l/kT$ (proportional to stress, σ, at high stress) and the value of n in the approximation $\dot{\epsilon} \propto \sigma^n$.

TABLE 2. COMPARISON OF THE ACTUAL AND THEORETICAL VALUES OF STRESS WHICH PRODUCE A GIVEN CREEP-RATE IN GOLD AT $T_\mathrm{m}/2$.

Predicted and actual values of n; theoretical value used for K.

actual, experimentally determined stress, σ	theoretical stress	$\mathrm{d}\ln\dot{\epsilon}/\mathrm{d}\ln\sigma = n$	
		theoretical	experimental
$10^{-3}\mu$	$0.96\times10^{-3}\mu$	5.0	5.5
$10^{-4}\mu$	$0.37\times10^{-4}\mu$	3.0	5.5

It seems that during dislocation creep in a two phase material the particles of the two phases deform like the specimen itself and so a simple law of mixtures, based on the deformability of the phases in isolation, often yields a good account of the creep rate of the composite.

DEFORMATION DURING TRANSFORMATION FROM ONE PHASE TO ANOTHER

An additional mechanism of deformation can operate in a two phase material if it is subjected to stress while in the course of actually transforming from one phase to another. Imagine that the transformation from beta to alpha occurs by the formation, in a matrix of beta, of a small region of alpha and let there be an increase in volume due to the phase change. Then transformation will cause an internal stress (I) and it will also deform the material (at rate $\dot{\epsilon}_i$ say). We can use these values to form an approximate estimate of the constant λ_{L} in the Levy–Mises stress–strain increment relations:

$$\dot{\epsilon}_{ij} = \lambda_{\mathrm{L}}\,\sigma_{ij}, \tag{39}$$

i.e.

$$\lambda_{\mathrm{L}} \approx \dot{\epsilon}_i/I. \tag{40}$$

Equation (39) with λ_{L} defined by equation (40) then becomes the creep equation for a material which is transforming. The condition for the applicability of equation (40) is seen to be that the second invariant of the strain tensor is not significantly altered by the external stress system. Or more simply:

$$\sigma \ll I. \tag{41}$$

Roberts & Cottrell (1958) were the first to suggest this type of creep and a more exact derivation of the equation was given by Anderson & Bishop (1962).

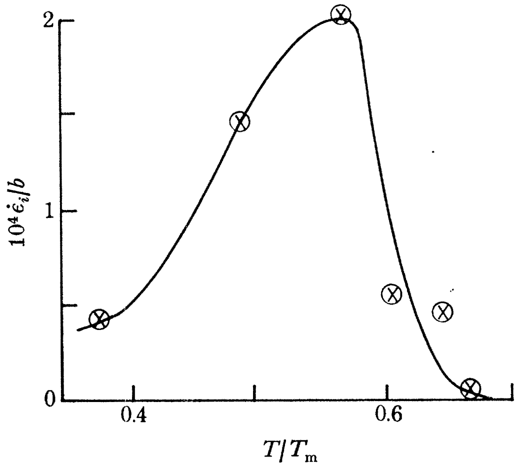


Figure 15. Dislocation-flux ($\dot{\epsilon}_i/b$) measurement for copper against homologous temperature (T/T_{m}) for a displacement-rate ($\dot{\phi}$) of 1.5×10^{-2} atom per lattice-site per second.

There is good support for the equation (Gittus 1975*b*). For example, it represents quite well creep in ferrous alloys during transformation and the creep of uranium during transformation. Indirect evidence indicates that the effect is also present during the transformation of dilute alloys of zirconium. In order to produce significant strains it is necessary to cycle the material through the phase change. If this is not done then creep by this mechanism ceases as soon as the transformation is complete. An exception is provided by the analogous case of irradiation creep in cubic materials such as stainless steels and nickel based alloys. Here atoms which have been displaced from their lattice sites by bombardment tend to drift towards edge dislocations, arriving there in somewhat greater numbers than do the vacant lattice sites from which the atoms were displaced. The surplus vacancies then form internal voids and as a result the material *swells*.

The process is akin to transformation in the sense that atoms are continually crystallizing onto dislocations. Unlike a normal transformation, however, it continues as long as irradiation continues and so the associated irradiation creep should continue also. Its magnitude should be given by equations (39) and (40) and as an estimate of the value of $\dot{\epsilon}_i$ we can use the experimentally measured value of the swelling rate, $\dot{S}$. Alternatively we can measure the rate of climb of the dislocations by time lapse photography in the million volt electron microscope (Gittus 1975*b*), a technique which is particularly attractive since we can arrange matters so that some of the electrons displace the atoms whilst others form the magnified image. Both approaches have been used and they provide good confirmation of equations (39) and (40). Thus figure 15 shows the value of $\dot{e}_i/b$ measured for copper in the electron microscope and figure 16 gives values for the internal stresses deduced from measurement of dislocation-bow in similar experiments. When these results are used in equations (39) and (40), they lead to predicted creep rates which are in the order of magnitude observed in in-pile creep experiments. Again from the results of such creep experiments and the rate of swelling (which occurred simultaneously) it is possible, via the theoretical equations, to calculate the internal stress. This has been done for two sets of creep data and (Gittus 1975*b*) the estimates so produced are close to the irradiated yield point of the material: arguably a reasonable estimate of the internal stress which operated in-pile.

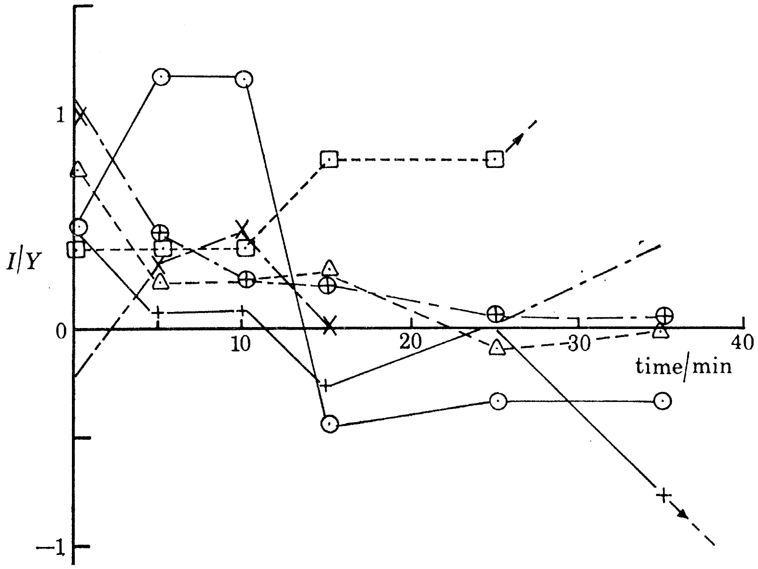


FIGURE 16. Internal stress (I) divided by the Young modulus (Y) deduced from measurements of dislocation bow at various instants.

DISCUSSION

In this paper we have drawn together the threads of contemporary understanding about the deformation properties of two phase materials. We see that these differ from single phase materials in several, on the face of it unexpected, ways.

First, there are the dispersion strengthened materials whose characteristic is that a tiny proportion of dispersed particles can produce an enormous reduction in deformability. The effect is explained by the way in which dislocations have to behave in order to circumvent the particles. Generally, to produce a significant strain rate, the dislocations must increase in length in order to get past the particles. The work which is needed to produce this increase in length is supplied by the external machine that produces the deformation. The smaller the strain (movement of the externally applied load) produced as a result of each bypassing event, the larger the external stress must be if it is to supply the work needed to increase the line length of the dislocation and thus permit it to bypass the particle.

The next class of materials which we have considered contain almost equal amounts of the two phases. They are the eutectics and eutectoids and they exhibit superplasticity under suitable conditions. We show that this is a process in which the particles of the two phases slide over one another like grains of sand in an hour-glass. Sliding is provisionally attributed to the glide of interphase boundary superdislocations and an equation which fits the available data and which is based on an analysis of this sliding process is presented. It contains a threshold stress which, it is shown, can reasonably be identified with the stress needed to overcome the friction between i.p.b. superdislocations and ledges in the interphase boundary. The high value of the activation energy for superplastic flow in two phase materials is seen to be in accord with recent measurements of i.p.b. self-diffusion coefficients.

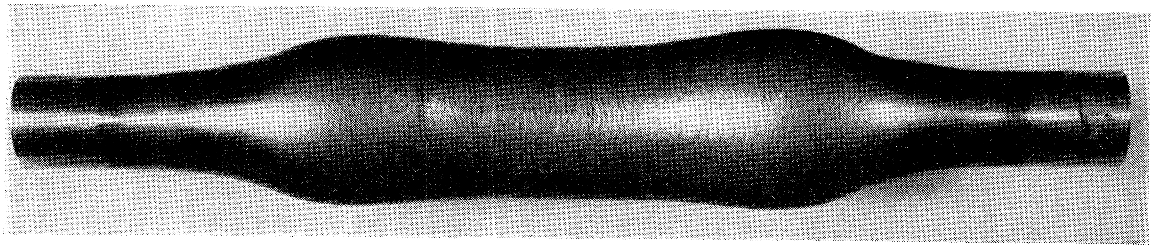


Figure 17. Specimen of the zirconium–tin alloy Zircaloy-2, illustrating the weakness and ductility which characterize superplasticity. The tubular specimen was heated by passage of a current of electricity: the ends were maintained colder than the centre by clamping them in water-cooled copper grips. Gas pressure was used to inflate the tube. The end regions were in the alpha-plus-beta phase field and swelled superplastically. The centre was in the beta phase-field and swelled by dislocation-creep: although much hotter than the ends it was evidently stronger (a consequence of the different deformation mechanisms).

If the strain rate is increased, then eventually within the grains (particles) there appears a network of dislocations arranged in cell–boundaries. The equation for creep in such a structure is developed. It invokes the movement of dislocations *across* the cells and so the cells shear during this type of creep ('dislocation creep') and the strain of the specimen (an aggregate of cells) equals that of an individual cell. In contrast, the dislocations which cause superplastic deformation were effectively trapped in the particle boundaries. So their glide gives rise, not to shear of the particle but to sliding on its boundary. It is this difference which accounts for the change from heterogeneous to homogeneous flow as we go from superplastic flow (at low strain rates) to dislocation creep (at higher strain rates).

The size of the cells increases, theoretically, as the applied stress is diminished. Experiments have confirmed this and agree numerically with the theory. The cell size is that which minimizes the free energy of the creeping specimen, by reducing both the external stress (needed to maintain a given creep rate) and the elastic energy of the dislocations themselves. The specimen is in a state of thermodynamic equilibrium and is resistant to perturbations such as those which lead to neck-propagation and fracture.

An even deeper minimum in the free energy occurs when the cell size becomes equal to the particle (grain) size and it is this which accounts for superplasticity – the property of extending by a factor of 2–20 without fracture (figure 17).

Finally we consider what happens if, during transformation from one phase to another, a material is subject to stress. This, it is shown, gives rise to an additional source of deformation. What happens is that the external stress perturbs slightly the conditions (high internal stress and internal deformation) set up by the transformation, leading to a component of external strain. The ratio of external stress to external strain rate is theoretically equal to the ratio of internal stress to internal strain rate (for small enough external stresses). Again it seems that experiments support the theory. In particular, when transformation is simulated by using irradiation to generate a constant super-saturation of self interstitials (which continually crystallize out on dislocations) then irradiation creep occurs at the theoretical rate and can be tied to measurements of transformation rate and internal stress made during irradiation in the high voltage microscope.

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Discussion

Alan H. Cook, F.R.S. (*Department of Physics, Cavendish Laboratory, Madingley Road, Cambridge* 0*B*3 0*HE*). It is very likely that the materials of the mantle of the Earth form not just two-phase but multiphase systems and so the ideas of strain hardening and superplasticity may be very important especially as the existence of a threshold strength could have significant consequences for our ideas of the topology of convection and of the conditions for its onset. Can Dr Gittus say how the phenomena scale in time and length? Will there still be a threshold strength when deformations extend over many millions of years and hundreds of kilometres?

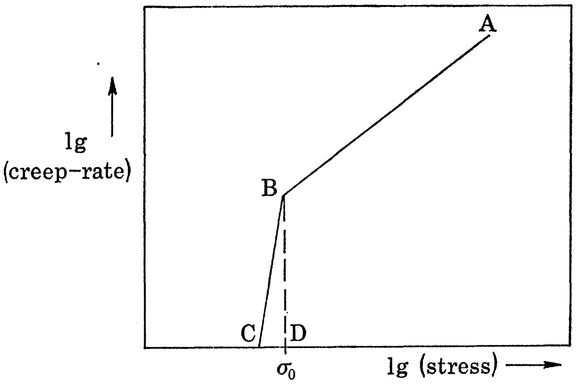

Figure 18. Plot of creep rate against stress illustrating the presence of a pseudothreshold stress σ_0.

J. Gittus. The situation is best understood by reference to figure 18 which is a plot of strain rate against stress on logarithmic axes. Line *ABD* corresponds to the case in which a threshold stress, σ_0, exists. Line *ABC* is the real case to which *ABD* is a good approximation for 'laboratory' timescales. However, when deformations extend over many millions of years and hundreds of kilometres the approximation is no longer adequate and portion *BC* shows that below the 'threshold stress' σ_0 there will still be slow but finite strain rates leading to finite displacements on geological timescales.

J. P. Poirier (*Saclay, France*). The equation Dr Gittus gives for superplastic deformation of two phase materials is adapted from the Ashby–Verrall equation for single phase materials, mainly by introducing a diffusion coefficient for heterophase boundary diffusion. Now, in the interface between A and B, we may define two diffusion coefficients for the diffusion of species A or B respectively. Which of these, or which combination of these, should be used?

J. Gittus. The derivation is an adaptation of Ball & Hutchison's (1969) model. It uses the Raschinger grain-switching process which was also used by Ashby & Verrall in their (somewhat different) model.

In cases (there is at the moment only one such case) where both diffusion coefficients are known, their average is used in the equation.

S. A. F. Murrell (*Department of Geology, University College London, Gower Street, London, WC1E 6BT*). Professor Weertman suggested that isostatic adjustment (due for example to the melting of an ice cap) takes place by transient creep processes, so that the mantle viscosity estimated from such data is a lower limit to the true viscosity. I have discussed the rôle of transient creep in lithosphere deformation in a recent paper (Murrell 1976). However, one of the problems in this field is that there is no recent general theory which relates transient creep to steady state creep, or defines the important parameters in transient creep. In my work I have used an early theory due to Mott (1953), but have also developed a new theory of transient creep based on Friedel's (1964) dislocation network growth theory (Murrell & Chakravarty 1973). Would Professor Weertman, Professor Ashby or Dr Gittus care to comment on the theory of transient creep?

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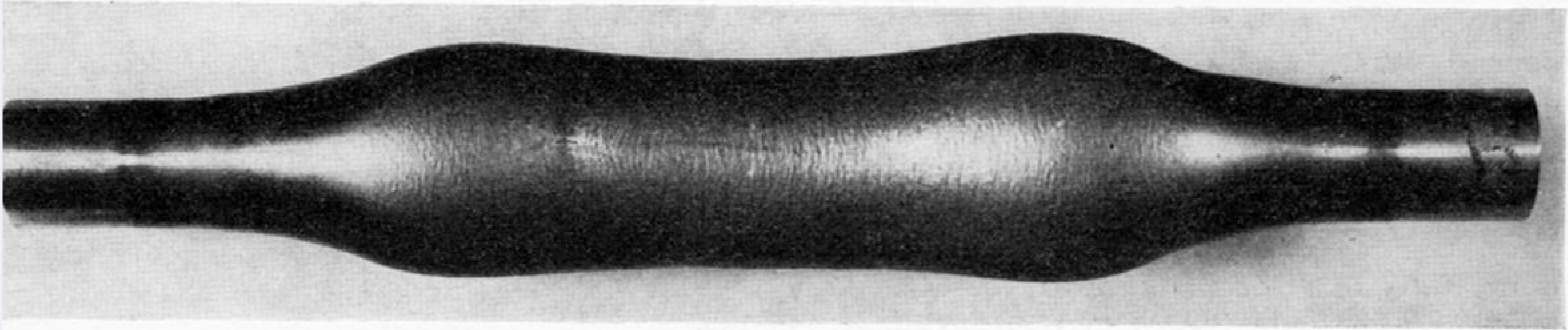
J. Gittus. A recent general theory which relates transient creep to steady state creep has been developed by me and is described in detail in my book (Gittus 1975*b*). See particularly § 3.3 on page 65 and equation 3.40 of that book.

J. Weertman. Obviously the transient creep field has not been developed, both experimentally and theoretically, to the extent that the steady-state creep field has been. Transient creep is much more complicated than steady-state creep because the deformational flow history of a sample can be very important. That is, if a sample is suddenly stressed its transient creep response will be different if the sample is well annealed or if the sample previously had undergone large plastic flow. Laboratory and theoretical investigations on transient creep phenomena that have been made generally are for the case of stresses suddenly applied to well annealed material. However, in glacial rebound, stress is 'suddenly' applied to mantle rock that presumably has been deformed to large plastic strains.



IGURE 17. Specimen of the zirconium–tin alloy Zircaloy-2, illustrating the weakness and ductility which characterize superplasticity. The tubular specimen was heated by passage of a current of electricity: the ends were maintained colder than the centre by clamping them in water-cooled copper grips. Gas pressure was used to inflate the tube. The end regions were in the alpha-plus-beta phase field and swelled superplastically. The centre was in the beta phase-field and swelled by dislocation-creep: although much hotter than the ends it was evidently stronger (a consequence of the different deformation mechanisms).